AN INVESTIGATION OF THE TRANSIENT HEAT

TRANSFER BETWEEN A TUBE AND

THE FLUID FLOWING THROUGH THE TUBE

A Thesis

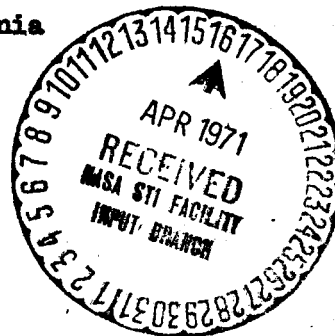Presented to

The Faculty of the School of Engineering

The University of Virginia

In Partial Fulfillment

of the Requirements for the Degree

Master of Aeronautical Engineering

by

Joseph H. Judd

June 1951

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CHAPTER I

INTRODUCTION

The computation of flow processes with heat trans-
fer has long been important in the field of engineering.
Until recent years nearly all problems of this type were
considered to have steady flow. The main objeetive of
research in this field was the determination of heat
transfer coefficients between the walls and the fluid and
the study of the mechanism of heat transfer through the
boundary layer and within the fluid. In modern times the
computation of transient heat transfer has become of im-
portance to the engineer designing heating-equipment. The
object of this paper was to develop methods for computing
the transient heat transfer between a fluid and its
boundaries.

The research for this paper was done at the Langley
Memorial Aeronautical Laboratory of the National Advisory
Committee for Aeronautics. The specific problem attacked
in this thesis was the computation of the transient
temperatures of an insulated tube, initially heated to a
constant temperature, and the fluid, at a constant en-
trance temperature, flowing through the tube. The results
of this investigation have several immediately useful
applications. The principal use is in the design of heat

exchangers of the heat accumulator type. The particular type of heat exchanger considered has banks of parallel tubes, initially heated or cooled, through which the working fluid flows. The methods presented in this paper enable the designer to compute the temperatures of the tube and the fluid for this heat exchanger.

Previous Work

A survey was made of the available literature on heat transfer to determine if any analytical work had been published on the transient heat transfer between a fluid and its boundaries. This search yielded no results. Thus the only references for this paper were experimental reports. These papers were limited to the design of specific pieces of equipment. This type of work is exemplified by a report of the Pittsburgh Des Moines Steel Company[1] on the design of a heat exchanger for the NACA. Tests were made on a copper coil, a single steel tube and a hexagonal bundle of steel tubes to determine the change of tube and air temperatures with time.

Organization

This paper is divided into two main sections. The

[1] L. Adams - Heat Accumulator and Exchanger for the National Advisory Committee for Aeronautics. Research Report 7636 Pittsburgh Des Moines Steel Co.

first section contains an analytical solution for the transient temperatures of an insulated tube, initially at a constant temperature, and the fluid flowing through the tube. From Fourier's law for the conduction of heat and the energy equations, an integro-differential equation for the temperature difference between the pipe and the fluid was derived with the axial distance and time as independent variables. The boundary conditions were obtained from physical considerations and a solution was obtained. From the expression for the temperature difference equations were derived for the tube and the fluid temperatures as functions of the axial distance and the time.

The second section of this thesis describes the method by which the analytical results were verified. A test setup was constructed that duplicated the conditions imposed on the analysis. This consisted of a long insulated tube, suitable instrumented, through which air was induced. A description of the equipment and instrumentation starts the second section of this paper. The test procedure is outlined and the accuracy of the tests determined. Then a description of the evaluation of coefficients and computations is given. It was found that good agreement exists between computed and experimental values of the tube and fluid temperatures.

LIST OF SYMBOLS

a   -   Cross sectional area enclosed by tube, sq. ft.

a_b -   Area of straight section of entrance bell, sq. ft.

A   -   Internal surface area of tube per unit length, sq. ft./ft.

c_H -   Specific heat of the fluid, BTU/(lb.)($^\circ$F)

c_m -   Specific heat of the tube, BTU/(lb.)($^\circ$F)

C   -   Functions of x and t (Table 1)

d_1 -   Internal diameter of the tube, in.

d_b -   Diameter of straight section of entrance bell, in.

d_o -   External diameter of tube, in.

δ -   Time required for the first particles of fluid to travel a distance x along the tube, sec.

g_s -   Specific gravity of the manometer alcohol

h   -   Heat transfer coefficient, BTU/(sq. ft.)(sec.)($^\circ$F)

H   -   Total head pressure, lb./sq. ft.

k   -   Conductivity of the fluid, BTU/(sq. ft.)(sec.)($^\circ$F/ft.)

K_f -   Ratio of the heat transfer to the heat capacity of the fluid, 1/sec.

K_m -   Ratio of the heat transfer to the heat capacity of the tube, 1/sec.

l   -   Length of the tube, in.

M   -   Height of the manometer fluid, mm.

N_u -   Nusselt's number, $\frac{hd}{k}$

p   -   Static pressure, lb./sq. ft.

p_b -   Static pressure at entrance bell, lb./sq. ft.

P_r - Prandtl's number, $\dfrac{c_p \mu}{k}$

ρ - Density of the fluid, slugs/cu. ft.

Θ - Temperature difference between the tube and the fluid, $^\circ F$

θ - Difference between the initial tube temperature and the air inlet temperature, $^\circ F$

q - Heat flux, BTU/sec.

R_N - Reynolds' number, Vd/ν

s - Constant in Laplace transform

t - Time, sec.

T_f - Temperature of the fluid, $^\circ F$

T_p - Temperature of the tube, $^\circ F$

T_{po} - Initial tube temperature, $^\circ F$

T_{fi} - Inlet fluid temperature, $^\circ F$

μ - Absolute viscosity, lb./(ft.)(sec.)

ν - Kinematic viscosity, sq. ft./ft.

w - Weight of fluid flow, lb./sec.

w_f - Weight of fluid per unit length of tube, lb./ft.

w_m - Weight of tube per unit length, lb./ft.

V - Fluid velocity, ft./sec.

V_b - Velocity in the entrance bell, ft./sec.

V_1 - Velocity at the tube inlet, ft./sec.

x - Axial distance along the tube, in.

CHAPTER II

MATHEMATICAL ANALYSIS

In this chapter the first section of this paper is
presented. The transient heat transfer between a tube and
the fluid flowing through the tube is analyzed and a
theoretical solution for the tube and fluid temperatures
is developed. The special case considered in this paper
is that where a fluid at a constant entrance temperature
flows through a tube at an initially constant temperature.
No heat is transferred across the outer wall of the tube;
effectively the tube is perfectly insulated. Thus both
tube and fluid temperatures change with x, axial distance
along the tube, and time, t. The problem is to determine
analytically expressions for the tube and the fluid
temperatures as functions of x and t.

To simplify the analysis, the assumption is made
that no radial temperature gradients exist in the tube or
in the fluid. For the fluid, turbulent flow provides a
very rapid transfer of heat radially. The conductivity of
the metal is large compared with the conductivity of the
fluid film so that the assumption of no radial gradient
of heat in the tube is justified. A further assumption is
that the transfer of heat axially along the tube by con-
duction may be neglected. The validity of this assumption

depends on the ratio of the heat conducted along the tube
to the heat transferred to the fluid.

Isolation of a Dependent Variable

Fourier's law for the conduction of heat is written:

$$dq = hA\theta \, dx.$$ 2.1

The equations for the conservation of energy are

$$dq = - w_m c_{pm} \frac{\partial T}{\partial t}p \, dx,$$ 2.2
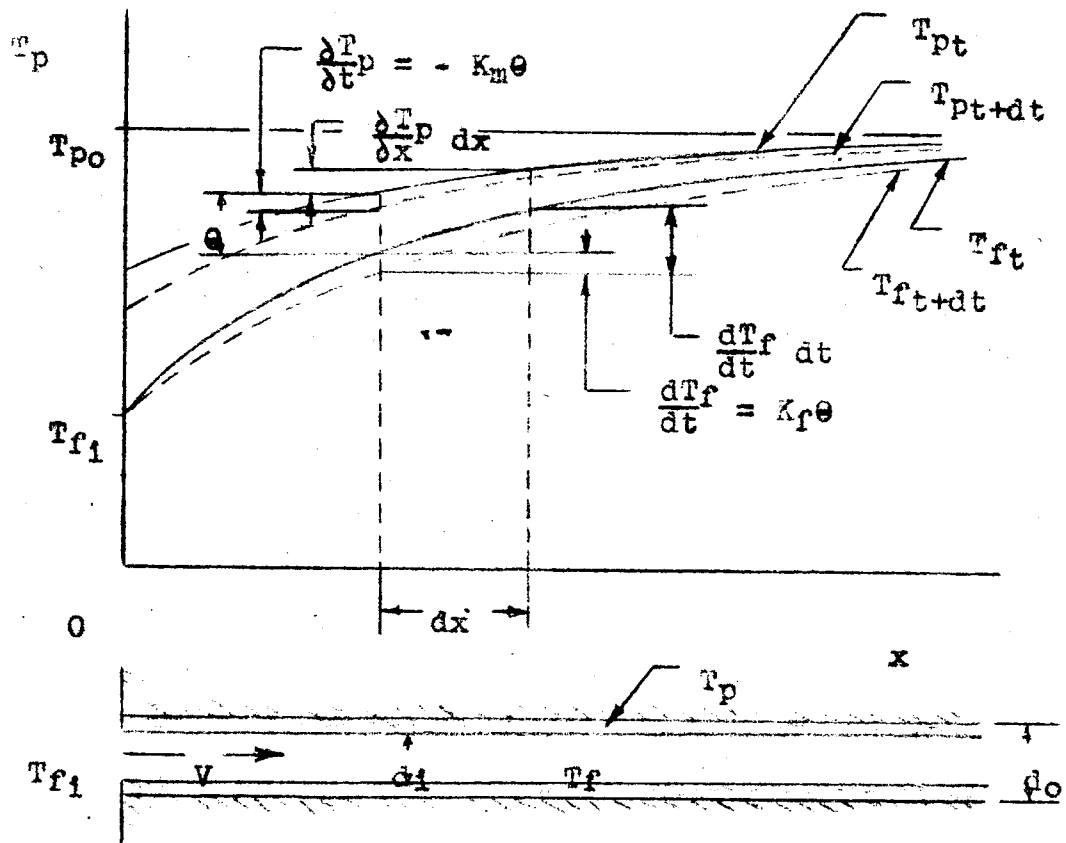

Figure 1.   Analytical conditions in the tube and the fluid
with initial tube temperature greater than
fluid inlet temperature.

and

$$dq = w_f c_{pf} \frac{dT_f}{dt} dx, \qquad\qquad 2.3$$

where q is the heat flux, h the turbulent heat transfer coefficient for fluids flowing through pipes, θ the temperature difference between the tube and the fluid, T_f and T_p the temperatures of the fluid and tube respectively, w_f and w_m the weight of the tube and fluid, and c_{pf} and c_{pm} the specific heat of the fluid and the tube respectively.

In order to reduce these equations to a single differential equation in terms of a common temperature function, the total differential of the temperature difference, $\theta = T_p - T_f$, is introduced.

$$d\theta = \frac{\partial \theta}{\partial x} dx + \frac{\partial \theta}{\partial t} dt , \qquad\qquad 2.4$$

or

$$d\theta = \left(\frac{\partial T_p}{\partial x} - \frac{\partial T_f}{\partial x} \right) dx + \left(\frac{\partial T_p}{\partial t} - \frac{\partial T_f}{\partial t} \right) dt. \quad 2.5$$

Equation 2.5 is rearranged and written

$$\frac{d\theta}{dt} = \frac{\partial T_p}{\partial x} \frac{dx}{dt} + \frac{\partial T_p}{\partial t} - \left(\frac{\partial T_f}{\partial x} \frac{dx}{dt} + \frac{\partial T_f}{\partial t} \right). \quad 2.6$$

The total derivative of the fluid temperature with respect to time is

$$\frac{dT}{dt}f = \frac{\partial T}{\partial x}f \frac{dx}{dt} + \frac{\partial T}{\partial t}f. \qquad 2.7$$

If equation 2.7 is substituted into equation 2.6, the derivative of θ with respect to t becomes

$$\frac{d\theta}{dt} = \frac{\partial T}{\partial x}p \frac{dx}{dt} + \frac{\partial T}{\partial t}p - \frac{dT}{dt}f. \qquad 2.8$$

From Fourier's equation for the conduction of heat, 2.1, and the conservation of energy equations, 2.2 and 2.3, the following relationships are obtained:

$$\frac{\partial T}{\partial t}p = - \frac{hA\theta}{w_m c_{p_m}}, \qquad 2.9$$

$$= - K_m \theta. \qquad 2.10$$

and

$$\frac{\partial T}{\partial t}f = \frac{hA\theta}{w_f c_{p_f}}, \qquad 2.11$$

$$= K_f \theta. \qquad 2.12$$

The terms K_f and K_m were introduced to simplify the expressions. These coefficients are assumed to be independent of temperature, i. e. constants, to linearize the differential equations. They represent the ratios of the heat transfer to the heat capacity of the tube and the fluid.

The partial derivative of the tube temperature may be written:

$$\frac{\partial T}{\partial x}p = \int_0^t \frac{\partial}{\partial x} \left(\frac{\partial T}{\partial t}p \right) dt \qquad 2.13$$

$$\frac{\delta T}{\delta x}p = - K_m \int_0^t \frac{\delta \theta}{\delta x}\, dt. \qquad\qquad 2.14$$

The derivatives of the tube and fluid temperatures, 2.10, 2.12 and 2.14 are substituted into equation 2.8.

$$\frac{d\theta}{dt} = - K_m \frac{dx}{dt}\int_0^t \frac{\delta \theta}{\delta x}\, dt - K_m\theta - K_f\theta. \qquad 2.15$$

Thus as integro-differential equation has been derived in terms of a single dependent variable, θ, and the independ- variables x and t. By using the relationship, $V = \dfrac{dx}{dt}$ and the total derivative of θ, equation 2.15 becomes

$$V \frac{\delta\theta}{\delta x} + \frac{\delta\theta}{\delta t} = - K_m V \int_0^t \frac{\delta\theta}{\delta x}\, dt - (K_m + K_f)\, \theta. \qquad 2.16$$

The solution of this differential equation for θ with the appropriate boundary conditions is the key to the analysis. Once θ has been determined the solution for the tube and the fluid temperatures is easily obtained.

Boundary Conditions

The boundary conditions are obtained from the physical conditions of the flow process. Before the flow starts, that portion of the fluid within the tube attains the same initial temperature as the tube. If the time at which the flow starts is called 0, this condition may be written:

$$\theta = 0 \qquad t = 0, \ 0 \leq x \leq 1. \qquad 2.18$$

The fluid is considered to be drawn from a reservoir at a constant inlet temperature, T_{fi}. Then at the entrance of the tube during flow

$$\theta (0,t) = T_p - T_{fi} \qquad 2.19$$

When the derivative with respect to t is taken and the value for the derivative of the tube temperature, 2.10, substituted,

$$\frac{d\theta(0,t)}{dt} = \frac{dT_p}{dt} \qquad 2.19$$

$$= - K_m \theta \qquad 2.20$$

The boundary condition for this differential equation is that at $t = 0$ the temperature difference equals a constant, $\theta_i = T_{po} - T_{fi}$. When this boundary condition is substituted into the solution of equation 2.20, the second boundary condition for the problem is obtained.

$$\theta = \theta_1 e^{-K_m t} \qquad t > 0, \ x = 0 \qquad 2.21$$

Solution of the Differential Equation

This section of the analysis deals with the solution of the boundary value problem which has been deriv-in the preceding sections. Restating the problem for convenience,

$$V \frac{\partial \theta}{\partial x} + \frac{\partial \theta}{\partial t} = - K_m V \int_0^t \frac{\partial \theta}{\partial x} dt - (K_m + K_f) \theta. \qquad 2.16$$

$$\theta = 0 \qquad\qquad t = 0, \; 0 \leq x \leq 1 \qquad 2.17$$

$$\theta = \theta_1 e^{-K_m t} \qquad\qquad t > 0, \; x = 0 \qquad 2.21$$

This problem is amenable to solution by methods of the Laplace transform. The transforms of equations 2.16 and 2.21 are

$$V \frac{d\vartheta}{dx} + s \vartheta - \theta(x,0) = - \frac{K_m V}{s} \frac{d\vartheta}{dx} - (K_m + K_f), \qquad 2.22$$

$$\vartheta(0,s) = \frac{\theta_1}{s + K_m} , \qquad 2.23$$

where ϑ the transformed temperature difference is a function of x and the Laplace constant s. When the first boundary condition is substituted into equation 2.22 an ordinary differential equation is obtained.

$$\frac{d\vartheta}{dx} = - \frac{s + K_m + K_f}{(1 + \frac{K_m}{s}) \frac{x}{V}} \vartheta. \qquad 2.24$$

The solution for this differential equation is

$$\vartheta = K e^{- \frac{s + K_m + K_f}{(1 + \frac{K_m}{s})} \frac{x}{V}} \qquad 2.25$$

When the transformed boundary condition, 2.23, is put into equation 2.25, the constant K is evaluated and the function ϑ determined.

$$\vartheta = \frac{\theta_1}{s + K_m} e^{- \frac{s + K_m + K_f}{(1 + \frac{K_m}{s})} \frac{x}{V}} \qquad 2.26$$

This function has no singularities or discontinuities so the inverse transform is easily made. The exponential term is split into its components as shown,

$$-\vartheta = \frac{\theta_1 e^{-s \frac{x}{V}}}{s + K_m} e^{-K_f \frac{x}{V}} e^{\frac{K_f K_m}{s + K_m} \frac{x}{V}}, \qquad 2.27$$

then the term $e^{\frac{K_f K_m}{s + K_m} \frac{x}{V}}$ is expanded into a series. The inverse transform of this function is as follows:

$$\theta = 0 \qquad\qquad\qquad t < \frac{x}{V} \qquad 2.28$$

$$\theta = \theta_1 e^{-K_f \frac{x}{V} - K_m (t - \frac{x}{V})} \left\{ 1 + K_m K_f \frac{x}{V} (t - \frac{x}{V}) + \frac{K_m K_f \frac{x}{V} (t - \frac{x}{V})^2}{2!^2} + \cdots + \frac{K_m K_f \frac{x}{V}(t - \frac{x}{V})}{n!^2} + \cdots \right\}$$

$$t \geq \frac{x}{V} \qquad 2.28$$

Equations 2.28 were differentiated and substituted into the differential equation 2.16. This procedure proved that equations 2.28 were a solution to the equation. The first boundary condition was satisfied by the form of solution given by the inverse Laplace transform. To check the second boundary condition, x was set equal to 0. This gives the expression,

$$\theta = \theta e^{-K_m t}, \qquad 2.21$$

and shows that the solution satisfies this boundary condition.

Tube and Fluid Temperatures

Once θ has been determined the tube and fluid temperatures may be determined from the initial equations. Although both tube and fluid temperatures may be computed directly from equations 2.10 and 2.12, the tube temperatures are much easier to compute than the fluid temperatures. The relationship for the tube temperature is written:

$$\frac{\partial T}{\partial t}p = - K_m \theta \qquad\qquad 2.10$$

If a fixed point on the tube is considered, the derivative can be changed from the partial to the total derivative. When this is done the equation is integrated to give the tube temperature.

$$\int_{T_{p_o}}^{T_p} dT_p = - K_m \int_{-\frac{x}{V}}^{t - \frac{x}{V}} \theta\, dt. \qquad 2.29$$

To simplify the integration the transformation is made that $\delta = t - \frac{x}{V}$. Then equation 2.29 becomes

$$T_p - T_{p_o} = - K_m \int_0^\delta \theta\, d\delta \qquad 2.30$$

Now when the value of θ from equation 2.28, with $t - \frac{x}{V}$ replaced by δ, is substituted into equation 2.30, the equation becomes ready for integration.

$$T_p = T_{p_o} - K_m \theta_1 \int_0^\delta e^{-K_f \frac{x}{V}} {}^{-K_m \delta} \left\{ 1 + K_m K_f \frac{x}{V} \delta + \frac{\left[K_m K_f \frac{x}{V} \delta\right]^2}{2!^2} + \right.$$

$$\left. \cdots + \frac{\left[K_m K_f \frac{x}{V} \delta\right]^n}{n!^2} + \right\} d\delta. \qquad 2.31$$

Upon integration the following expression for the tube temperature is found:

$$T_p = T_{p_o} - \theta_1 e^{-K_f \frac{x}{V}} \left[1 - e^{-K_m \delta} + K_m K_f \frac{x}{V} C_1 + \frac{\left[K_f K_m \frac{x}{V}\right]^2 C_2}{2!^2} \right.$$

$$\left. + \cdots + \frac{\left[K_f K_m \frac{x}{V}\right]^n C_n}{n!^2} + \right], \qquad 2.32$$

where C_n are functions of δ and given in table 1. From equation 2.32 the temperature of the tube may be computed at any station and at any time.

TABLE I

TUBE TEMPERATURE COEFFICIENTS	
	Value
C_1	$-\dfrac{e^{-K_m \delta}(K_m \delta + 1)}{K_m} + 1/K_m$
C_2	$-\delta^2 e^{-K_m \delta} + 2C_1/K_m$
C_n	$-\delta^n e^{-K_m \delta} + nC_{n-1}/K_m$

The temperature of the fluid is computed from the definition of the temperature difference θ.

$$T_f = T_p - \theta.$$ 2.33

The tube temperature is computed from equation 2.32 while θ is computed from 2.28.

CHAPTER III

EXPERIMENTAL APPARATUS

Mechanical Equipment

To provide a check for the applicability of the
analytical results, an experimental setup was devised to
fulfill the conditions of the analysis. A long tube was
instrumented with thermocouples and preheated to an
initially constant temperature. Heat transfer to or from
the outer surface of the tube was lowered by the use of
insulation. Then air was drawn through the tube and the
tube and air temperatures recorded. The general experiment-
al layout is shown in block diagram form in figure 2. To
ensure a uniform flow of air at a constant temperature and
pressure the air was induced through the tube by a blower.
Since the temperatures were continually varying, recording
potentiometers were used for temperature measurements.

The tube that was used in these tests was a 5/16-
inch hard copper tube, eight feet long, with an inside dia-
meter of .2435 inches. Figure 3 shows the details of the in-
let and exit of the tube. A mahogany entrance bell with a
contraction ratio of 54.8 and a 5/16-inch diameter straight
section was used as an inlet. Since the inside diameter of
the tube is smaller than the adjoining straight section of

the entrance bell, the flow has to pass over a sharp corner as it enters the tube. The exit of the tube was faired into a mahogany diffuser that had a 20° expansion from a .2435-inch diameter to a 1.5-inch diameter. A valve was located at the end of the diffuser to control the air velocity through the tube. A flexible hose connected this valve to the blower. The blower was of the centrifugal type, electrically driven.

The tube was placed in a 2½ by 3-inch balsa wood beam for insulation and suspension in the heating box. A sealed air space of 7/16-inch diameter separated the tube from the beam. The tube was supported from the beam by five equally spaced wooden washers. The balsa beam was flanged at each end for connection to the entrance bell and the diffuser block. Rubber gaskets were placed between the flanges and the block faces to provide air tight seals.

To bring the tube up to the desired temperature, the tube in its insulating beam was suspended inside a heating box. The box, 96 × 12 × 12 inches inside dimensions, was constructed of wood-framed celotex sheet. Nichrome wire, spaced along the box in such a way that the tube had a constant temperature, was used for the heating element. The temperature of the box was controlled by a Fenwall

thermoswitch which was preset to the desired temperature.

Instrumentation

The static pressure of the flow through the entrance bell was measured one diameter from thr reduction section and two diameters from the tube entrance. This pressure was measured on a Wallace-Tierney electrically driven micromanometer. Alcohol with a specific gravity of .829 was used for the manometer fluid.

The temperatures of the air and the tube were measured by thermocouples. Table II gives the location of these thermocouples measured from the tube entrance. Iron-constantan thermocouples, 36 gage, were used to measure the air temperatures. The thermocouple beads were two to five hundredths-inch in diameter and located along the centerline of the tube. This gives an effective immersion of 24 diameters. The lag of the thermocouples was of the order of one half second. Copper-constantan thermocouples were used to measure the tube temperatures. The copper tube served as one side of the thermocouple while constantan wire, 30 gage, was used for the other side. The constantan wire was peened into the center of the tube to form the metallic joint. This type of installation has no time lag in temperature measurement. Calibration of the

thermocouple material indicated an accuracy of $\pm$ .25 per-cent.

The temperatures were recorded by two single channel Brown self-balancing potentiometers with automatic cold junction compensation. These recorders had a 0 - 6 milli-volt range and a basic accuracy of $\pm$ .33 percent. A motor driven Lewis switch was used to switch thermocouple leads and transmit the individual potential into the recorders. To simplify reading the records, the air temperatures were recorded on one potentiometer and the tube temperatures on the other.

CHAPTER IV

TESTS AND MEASUREMENTS

Procedure

The tube was heated by placing the tube and its
insulating beam in the heating box. After power to the
heating elements was turned on, the tube was soaked at
constant box temperature until the desired operating
temperature was reached. During the heating period both
the entrance and exit of the duct were closed to aid in
obtaining a uniform temperature distribution along the
tube. Checks on the uniformity and magnitude of the temp-
eratures along the tube were made by operating the record-
ers and observing the temperatures. The variation of the
tube temperatures with axial distance along the tube is
shown in figure 5. This figure shows that the temperatures
are constant over most of the tube length and only vary
slightly at the ends.

Immediately before each run the duct was opened and
the recorders started. Then the air valve was set to attain
the desired inlet velocity and the blower started. The air
velocity was held constant by manual control of the air
valve with the micromanometer as a pressure reference.

To investigate the effect of temperature magnitude,
tests were made at three initial tube temperatures, 132,

167 and 227 degrees Fahrenheit. The velocity for each of these runs was held constant at 103 feet per second.

The tube and air temperatures at each station were plotted against time by the recording potentiometers. Sample records of this data are shown in figure 4. The static pressure in the entrance bell and the room temperatures were noted by the operator.

Experimental Accuracy

The accuracy of the individual components of the temperature measurement system was given in chapter III. The thermocouple material had an error of $\pm$ .25 percent and the recorders $\pm$ .33 percent. Thus mechanically the error totaled to $\pm$ .58 percent. A further source of error occurs in reading the temperature records. This is estimated to be $\pm$ .5 degree Fahrenheit. The experimental error in temperature measurement therefore varies from $\pm$ 2.25 percent for the low temperatures to $\pm$ .92 percent at the high temperatures.

Determination of the Flow Velocity

Since the temperatures were measured directly, the only computation necessary was that for the initial air

velocity. Since the velocity of the air in these tests was small compared with the speed of sound, Bernoulli's equation for incompressible flow was used to relate the flow quantities.

$$p_b + \tfrac{1}{2} \rho V_b^2 = H, \qquad\qquad 4.1$$

where H is the barometric pressure, ρ the air density and V_b the velocity at the entrance bell. This may be rewritten in the form

$$V_b = \sqrt{\frac{2 \left(H - p_b \right)}{\rho}} \qquad\qquad 4.2$$

Now $H - p_b$ represents the differential, measured by the micromanometer, pressure between the room and the straight section of the entrance bell. If M represents the height of the manometer column in millimeters of alcohol and g_s the specific gravity of the alcohol, the expression, 4.2, becomes

$$V_b = \sqrt{\frac{.0409\ g_s M}{\rho}} \qquad\qquad 4.3$$

The velocity of the fluid entering the tube was obtained from the continuity equation.

$$\rho_b a_b V_b = \rho a V_1, \qquad\qquad 4.4$$

where a represents the area of the respective sections. Now for an incompressible fluid ρ is a constant and the expression for the inlet velocity is written:

$$V_1 = \sqrt{\frac{.0409 \ Mg_s}{\rho}} \ \left(\frac{d_b}{d_1}\right)^2 \qquad\qquad 4.5$$

CHAPTER V

COMPUTATION PROCEDURE

In this chapter the procedure used to compute the tube and fluid temperatures for conditions of the experiment are outlined. The first problem that must be faced in this computation is the evaluation of the coefficients K_f and K_m. Then methods are outlined for computing the tube and air temperatures to take best advantage of the analytical results.

Evaluation of Coefficients

It was mentioned in Chapter II that the analytical results are best applied when turbulent flow exists in the tube. By making the flow pass over a sharp corner as it enters the tube, the flow becomes turbulent very close to the entrance of the tube. Krieth and Summerfield[1] have showed that turbulent flow exists 3.42 diameters downstream for this type of entrance. Thus the use of the turbulent heat transfer coefficient over the entire tube length seems justified. The value of the turbulent heat transfer coefficient was taken from McAdams.[2]

[1]Frank Krieth and Martin Summerfield. Investigation of Heat Transfer at High Flux Densities (CIT, Pasadena, Guggenheim Aeronautical Lab., Jet Propulsion Lab. PR4-65, 1948)

[2]William H. McAdams, Heat Transmission, (New York, McGraw-Hill Book Company, 1942). p. 168.

$$N_u = 0.023 \ R_N^{.8} P_r^{.4}, \qquad 5.1$$

or rewriting

$$h = 0.023 \ \frac{k}{d} \ R_N^{.8} P_r^{.4}, \qquad 5.2$$

where the conductivity, k, the Reynolds number, R_N, and the Prandtl number, P_r, are all functions of the fluid temperature.

The heat transfer coefficient can be computed by using either the inlet fluid temperature, an average value of fluid temperature or that obtained by an iterative process. In this paper the initial fluid temperature was used as a start and the agreement obtained between the computed and experimental values was so good, considering the limitations of the experiment that no further work was done. In any case the change in heat transfer coefficient through the tube was only 12 percent for the highest temperature range tested.

The values of Prandtl number, specific heat, viscosity, and conductivity of the fluid were taken from Keenan and Kaye[1]. The Reynolds number was computed from the definition.

$$R_N = Vd_1/\iota, \qquad 5.3$$

where ι is the kinematic coefficient of viscosity.

[1] Joseph H. Keenan and Joseph Kaye, Gas Tables, (New York, John Wiley and Sons, Inc., 1948).

The next step is the evaluation of the coefficients K_f and K_m. These were defined to be

$$K_f = \frac{hA}{w_f c_{pf}} ,$$ *5.4*

and

$$K_m = \frac{hA}{w_m c_{pm}} ,$$ 5.5

It should be emphasized at this point that A is the surface are per unit length and w_m and w_f are weight per foot.

The variation of K_m and K_f along the tube at a time of ten seconds is plotted in figure 6. K_f changes 37 percent over the length of the tube for an initial tube temperature of 227 degrees Fahrenheit while K_m has only the 12 percent change caused by the variation of the heat transfer coefficient. The compressibility effect causes the large change in K_f with temperature since the weight of fluid changes rapidly as the temperature changes.

Computation of the Tube and Air Temperatures

Using the values of K_m and K_f in equations 5.4 and 5.5, the tube and air temperatures can be computed from equations 2.32 and 2.33. However this method of computing the tube temperatures is very laborious since the series in equation 2.32 does not converge rapidly.

To perform rapid computations of tube and air temperatures, the summation of the series in equation 2.28 calculated and plotted against the series term $K_f K_m \frac{x}{V}(t- \frac{x}{V})$. Once this has been done θ is easily computed using values from this curve in equation 2.28. From equation 2.29 of chapter II the following equation is obtained:

$$T_p = T_{p_0} - K_m \int_{\frac{x}{V}}^{t - \frac{x}{V}} \theta \, dt. \qquad 5.6$$

Now the integral in the last term can be evaluated graphically in a short time. θ, for a station on the tube, is plotted against time. The area under the curve represents the value of the integral.

In a similar manner an equation for the fluid temperature may be written.

$$hA\int_0^x \theta \, dx = c_{p_f}w \left(T_{f_x} - T_{f_i} \right) \qquad 5.7$$

or

$$T_{f_x} = T_{f_0} + \frac{hA}{c_{p_f}w}\int_0^x \theta \, dx \qquad 5.8$$

In this case θ is plotted against x, the distance along the tube axis. The value of the integral is represented by the area under the curve.

CHAPTER VI

RESULTS AND DISCUSSION

Comparison Between Experimental and Computed Values of θ/θ_1

The theoretical and experimental values of θ/θ_1 are
plotted in figure 7. The qualitative agreement appears to
be good with one significant exception. These are the
values of θ/θ_1 in the vicinity of the tube entrance. If the
boundary condition, $x = 0$, is considered, it can be seen
that the analysis predicts that θ/θ_1 will assymtotically
approach zero as t becomes large. However the experimental
values tend to remain at some value above zero as the time
becomes large. This deviation on the part of the experiment-
al values occurs because heat is transferred to this end
of the tube from two sources, the tube itself and the in-
sulation around the tube. The ratio of heat transferred
to the fluid to the heat conducted along the tube runs about
thirteen to one at a time of ten seconds when a very steep
gradient along the tube exists. So it would seem that the
heat transferred from the insulation is pretty large. This
is most noticable at the tube entrance since the temperature
gradient radially from the tube becomes large rapidly.

Tube and Air Temperatures

The experimental values of tube and air tempera-
tures are shown in figure 9. These are in the form of
curves of temperature plotted against axial distance along
the tube for constant values of time. The only value of
temperature which is in serious disagreement with the fair-
ed curve is that indicated by the number two air thermo-
couple. The probable explanation of this discreprancy is
that the thermocouple was bent during installation and lies
close to the tube wall. The computed values of tube and air
temperature are plotted in figure 8. The shape of these
curves agrees well with the experimental plots but in mag-
nitude there is a moderate amount of discrepancy. The
computed values of tube temperature drop more rapidly than
the experimental points, especially at the tube entrance.
This is due to heat transfer from the insulation to the
tube. The air temperatures during the time shown on these
plots indicates good agreement both quantatively and
qualitatively.

In figure 10 a comparison is made between the com-
puted and experimental values of the temperature of the
air leaving the tube for the three runs. These show very
good agreement.

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Limitations of the Investigation

The major error introduced into this analysis holds
true only for gaseous fluids. This is the assumption that
the fluid is incompressible and is well illustrated by the
variation of K_f as shown in figure 6. Even so the range of
this investigation is where the compressibility effects
are small. If the temperature is increased by a large
factor or the velocity increased, it would be expected that
there would be large discrepancies between computed and
real values. To account for compressibility will make the
differential equation non-linear and introduce the greatest
difficulty in solving the problem.

The largest experimental error has been mentioned
previously in this chapter. That is the transfer of heat
from the insulation to the tube. The value of improving
the experimental apparatus seems doubtful in view of the
cost of further refinement. The major improvement would
be to seal the tube in an evacuated flask. However the
agreement between the computed and experimental results
was good enough to validate the analysis and that of course
was the major justification for the experiment.

Extension of the Problem

The next step in this type of investigation would

be to extend the analysis to include different boundary
conditions. There would be two major extensions; one, the
inlet air at some arbitrary function, and two, an arbitrary
temperature distribution along the tube.

CHAPTER VII

CONCLUSIONS

An analysis was made of the transient heat transfer between a tube and the fluid flowing through the tube. Expressions were obtained for the tube and fluid temperatures as functions of the independent variables x and t. An experiment was set up to verify the results of the analysis. Good agreement was obtained between the experimental and computed values of tube and air temperatures.

In previous computations, the tube and air temperatures were computed by a step-by-step integration of the equation for heat conduction. This process required weeks of computings for a single condition. The methods presented in this paper cut the computation time down to several days.

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4. Adams, L., Heat Accumulator and Exchanger for the National Advisory Committee for Aeronautics, (Pittsburgh Des Moines Steel Company, 1946).

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TABLE II

THERMOCOUPLE LOCATION

Air thermocouples		Tube thermocouples	
Number	Distance along the tube in inches	Number	Distance along the tube in inches
1	0	1	0.125
2	4	2	4.125
3	10	3	10.125
4	18	4	18.125
5	28	5	28.125
6	38	6	38.125
7	48	7	48.125
8	58	8	58.125
9	68	9	68.125
10	78	10	78.125
11	96	11	95.875
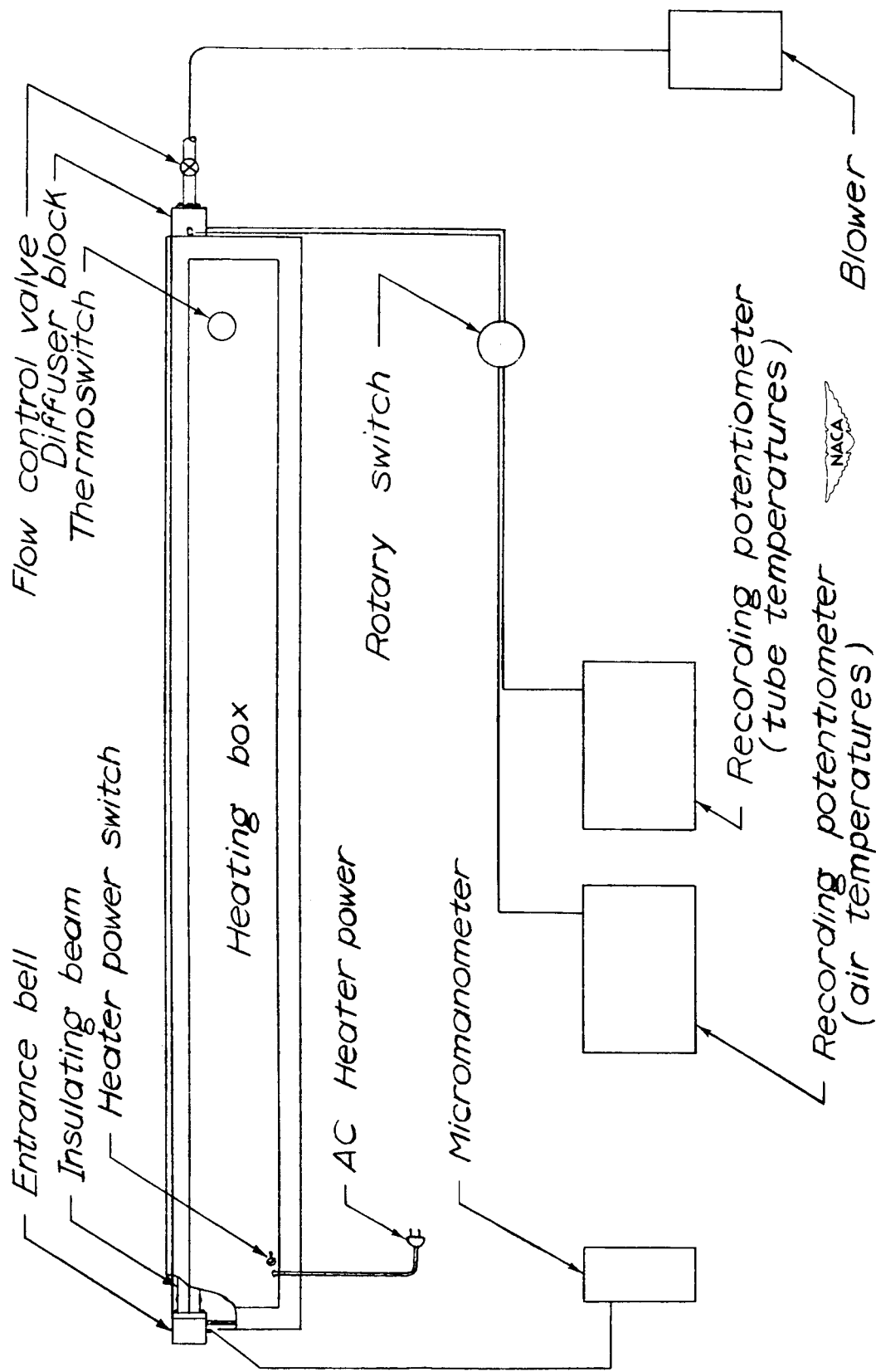
Figure 2.- Block diagram of experimental apparatus and instruments.

Entrance bell

Insulating beam

Air space

Static pressure
orifice

$\frac{5}{16}$ Copper tube

Diffuser

W — 96

6

3

Glyptol seal

Constantan wire peened
into center of tube wall.

$\frac{1}{8}$

Copper tube

Iron-constantan thermocouple

Details of thermocouple installation
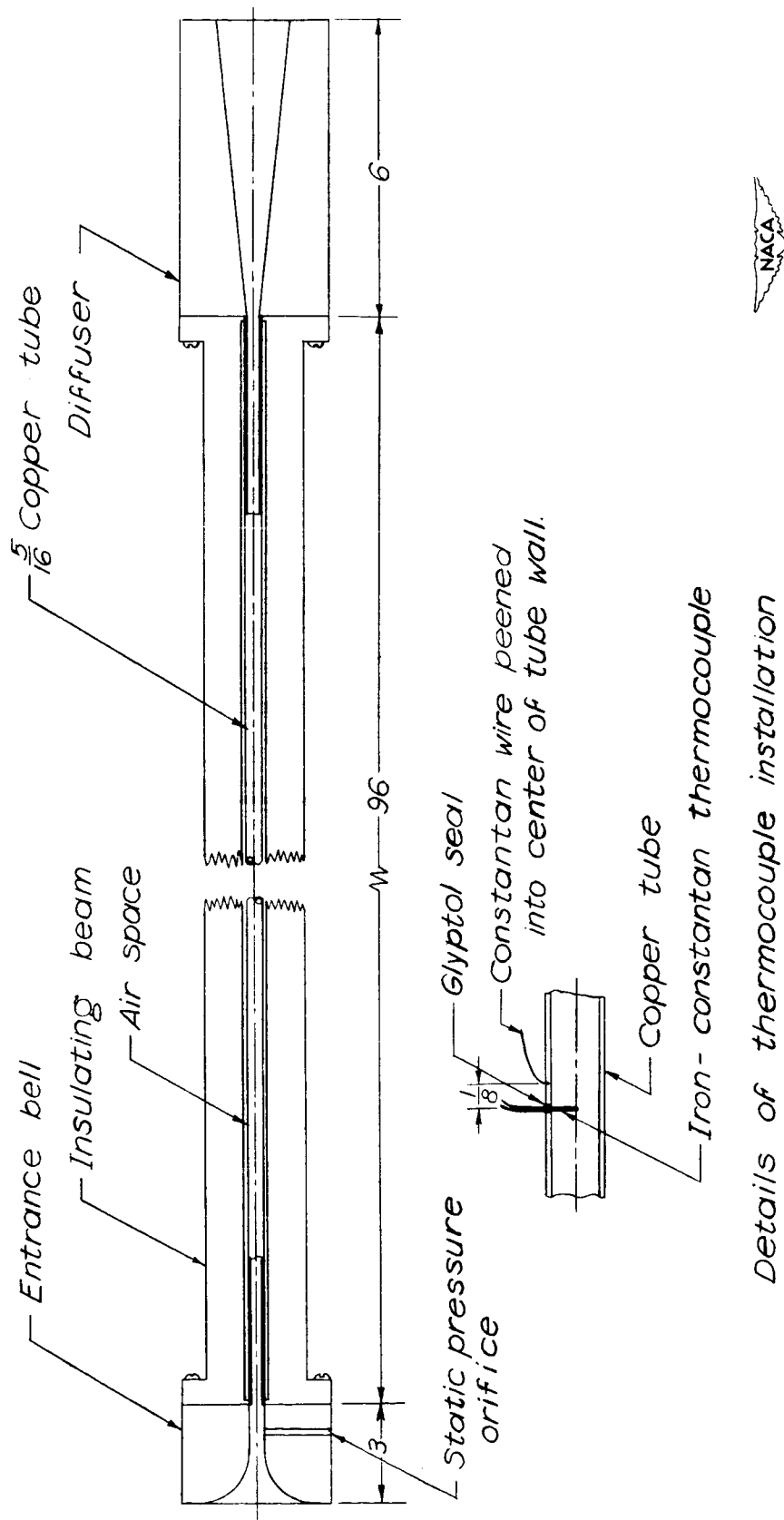
All dimensions in inches

NACA

Figure 3.- Installation and instrumentation of tubes.
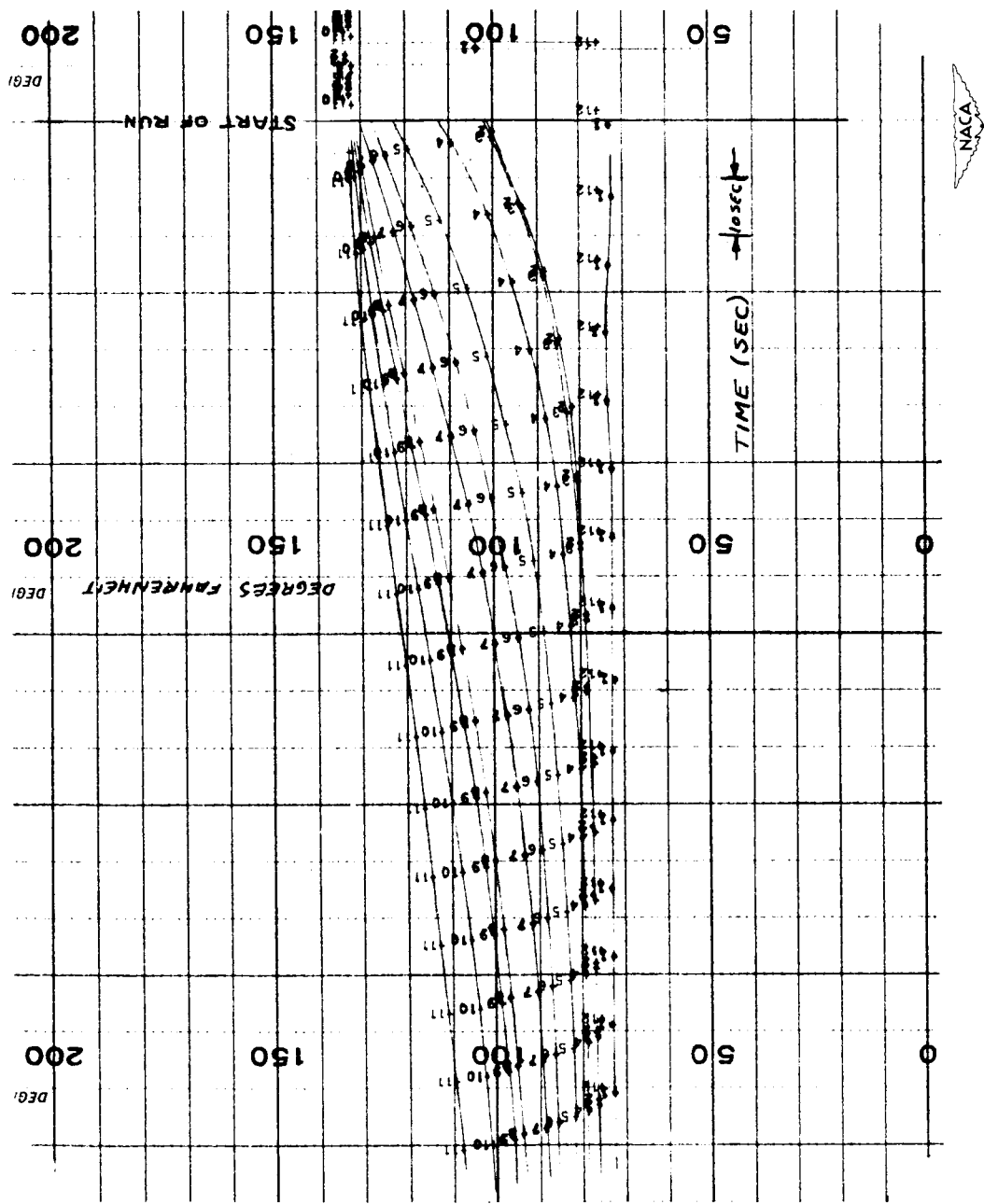
(a) Air temperatures.

Figure 4.- Sample records.
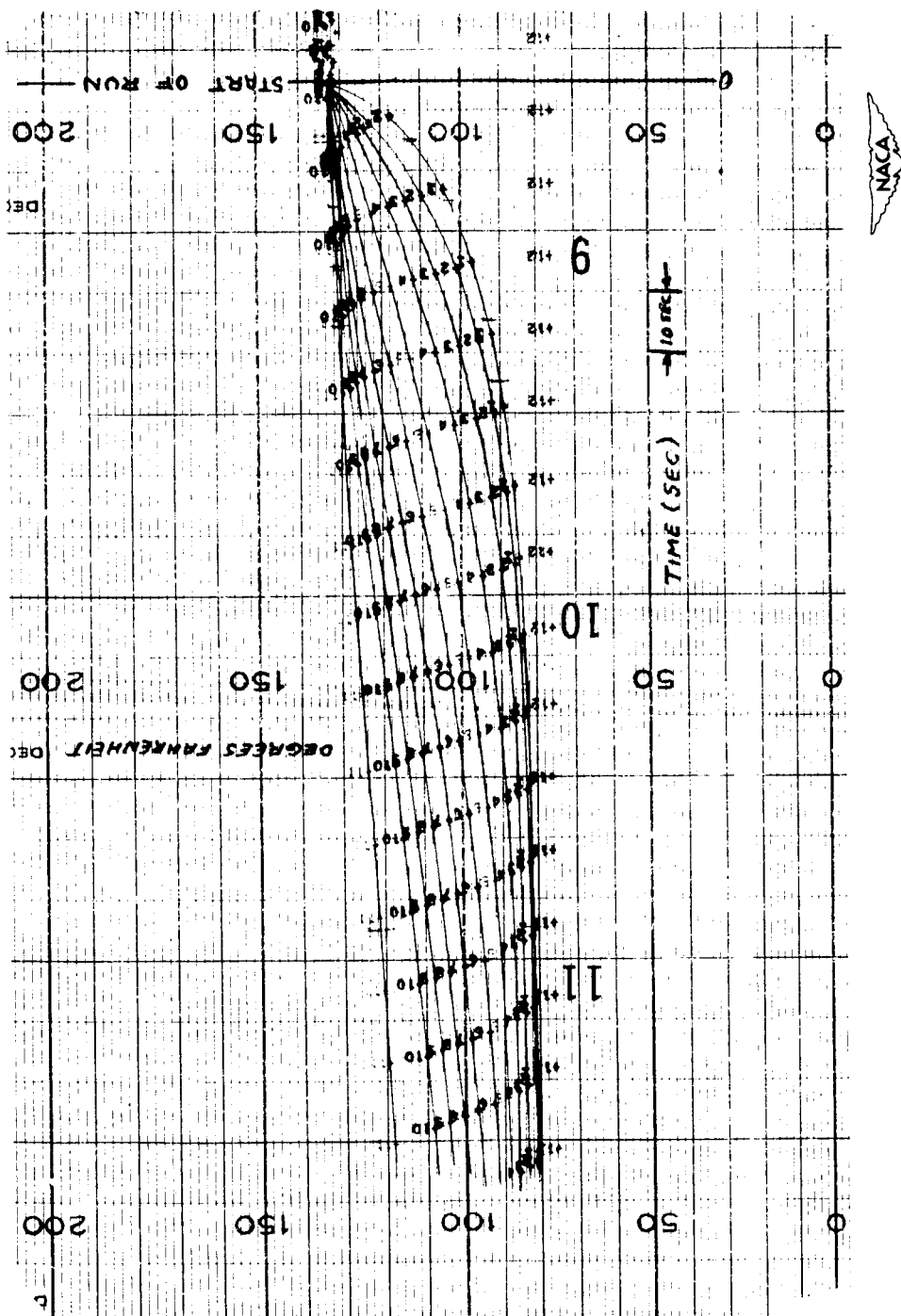
(b) Tube temperatures.

Figure 4.- Concluded.
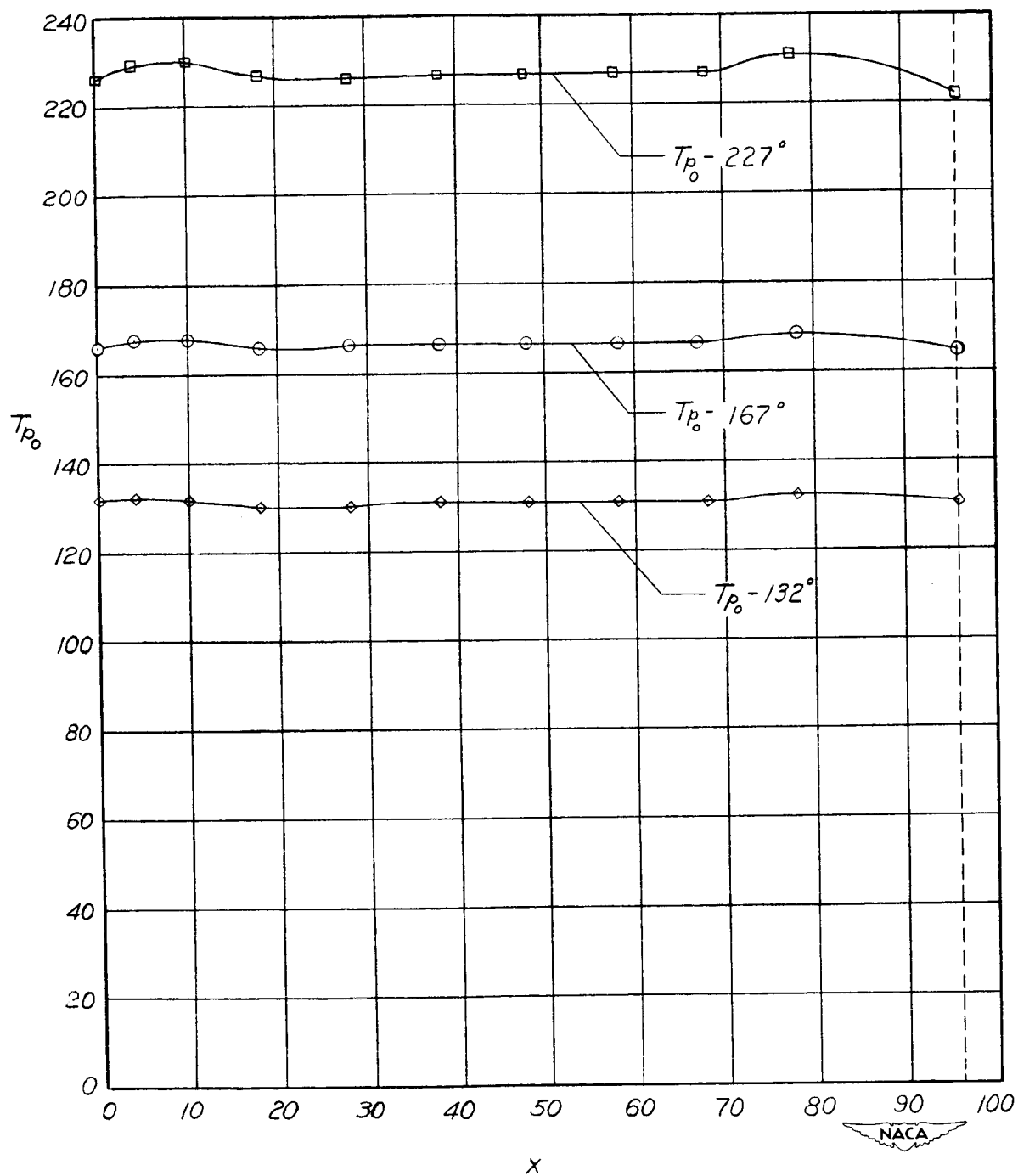
Figure 5.- Variation of initial tube temperatures along the tube.
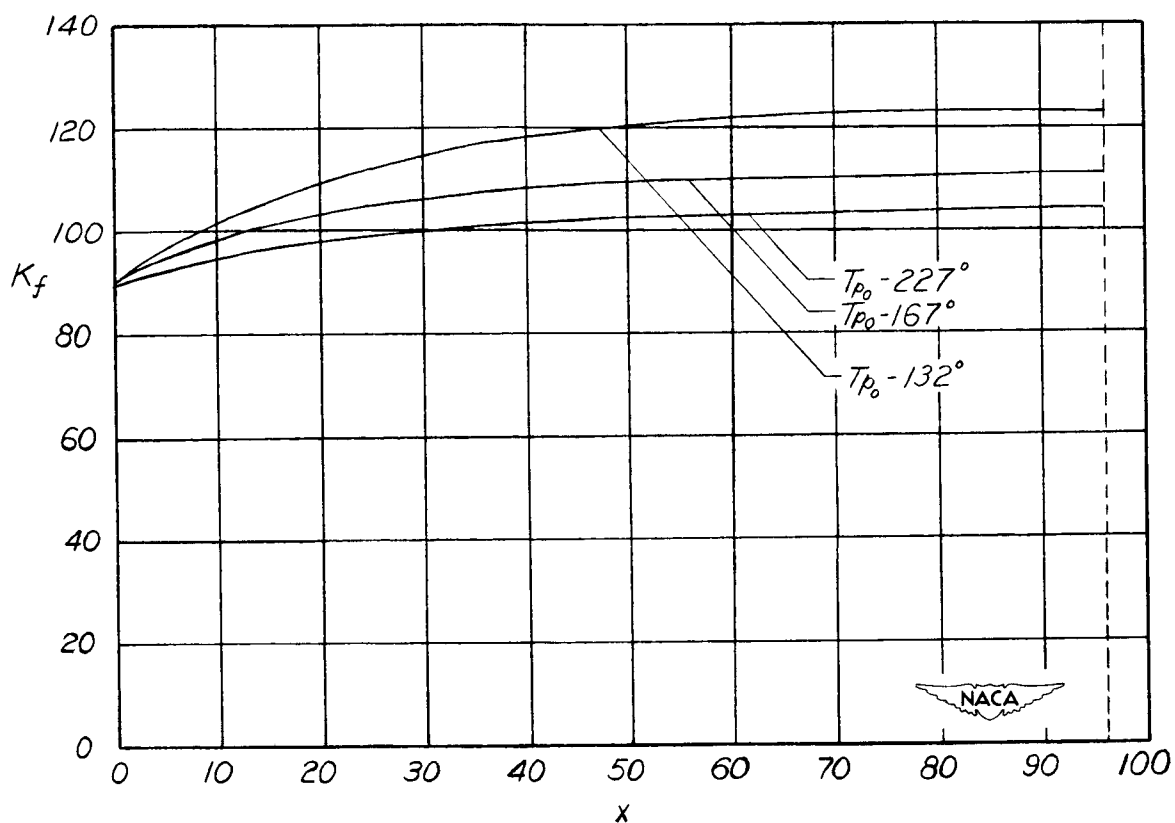
(a) Variation of K_f with x.
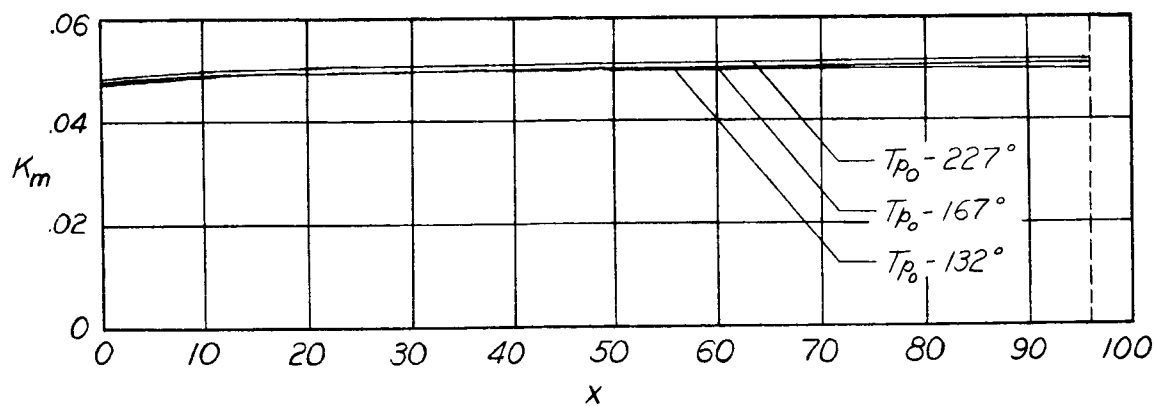
(b) Variation of K_m with x.

Figure 6.- Variation of coefficients K_f and K_m along the tube at 10 seconds.

Experimental values of $\frac{\theta}{\theta_i}$.
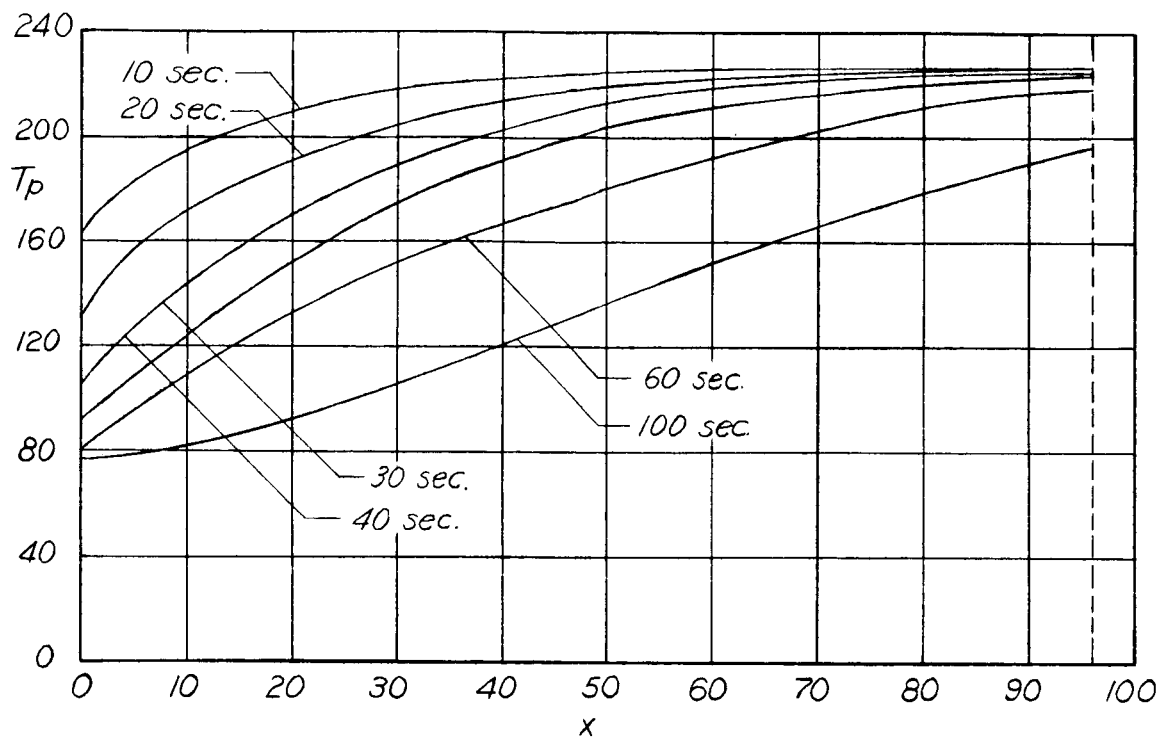

Computed values of $\frac{\theta}{\theta_i}$.
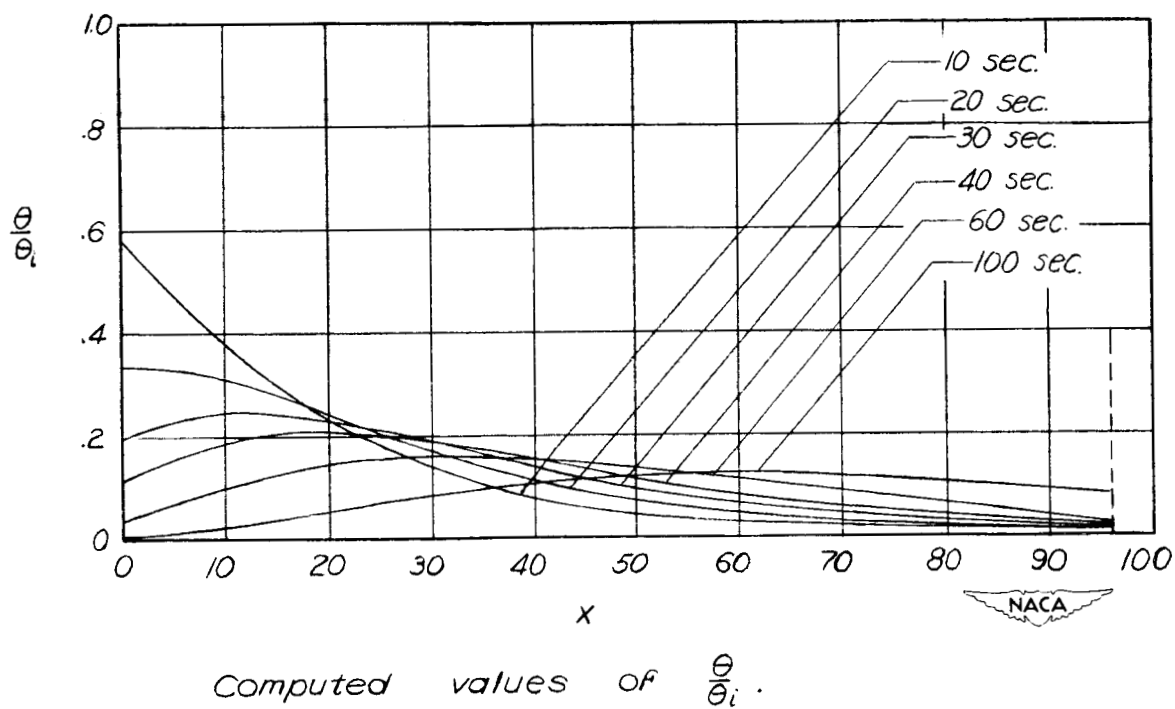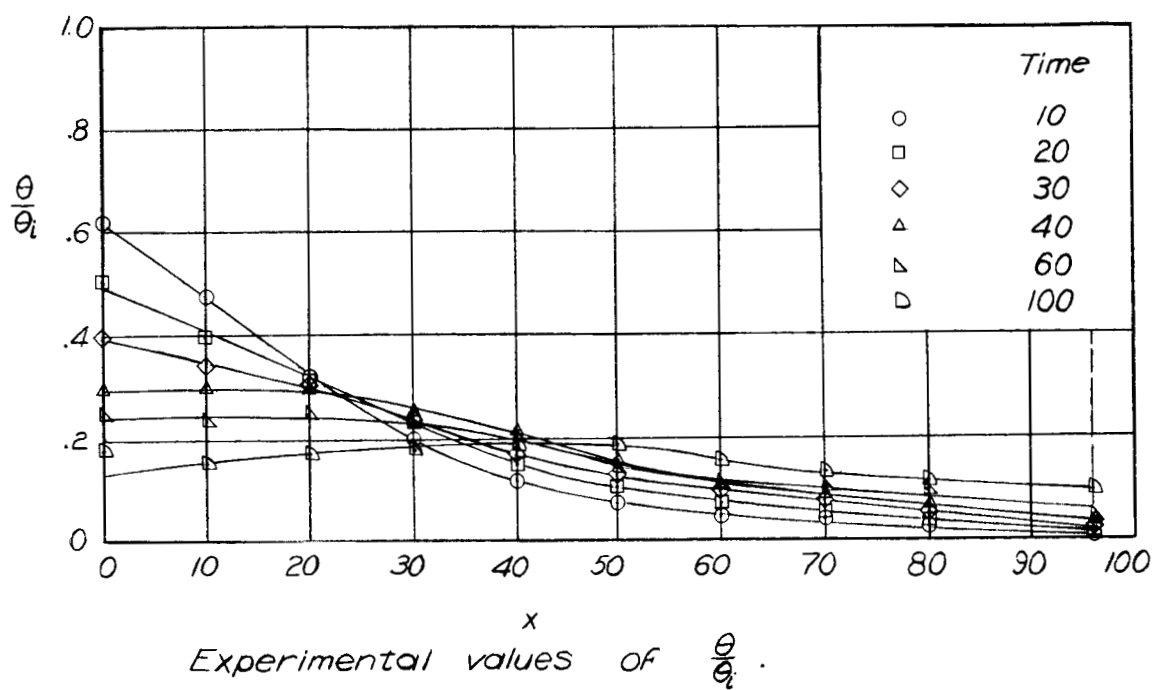
(a) Initial tube temperature of $227°$.

Figure 7.- Experimental and computed values of θ/θ_i.
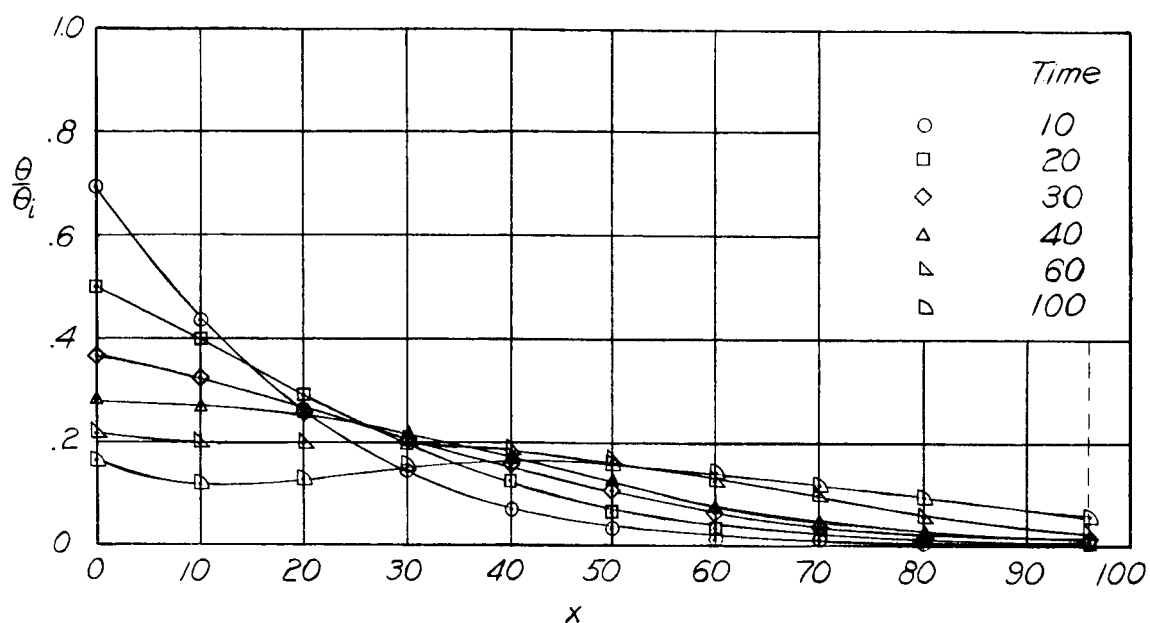
Experimental values of $\frac{\theta}{\theta_i}$.
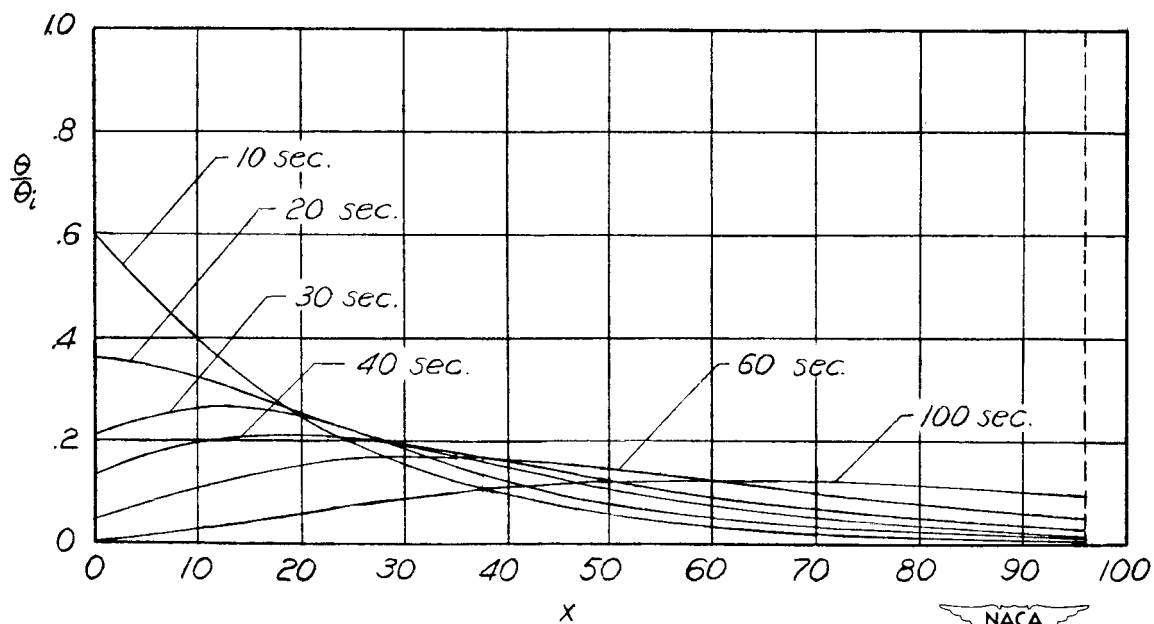
Computed values of $\frac{\theta}{\theta_i}$.

(b) Initial tube temperature of 167°.

Figure 7.- Continued.
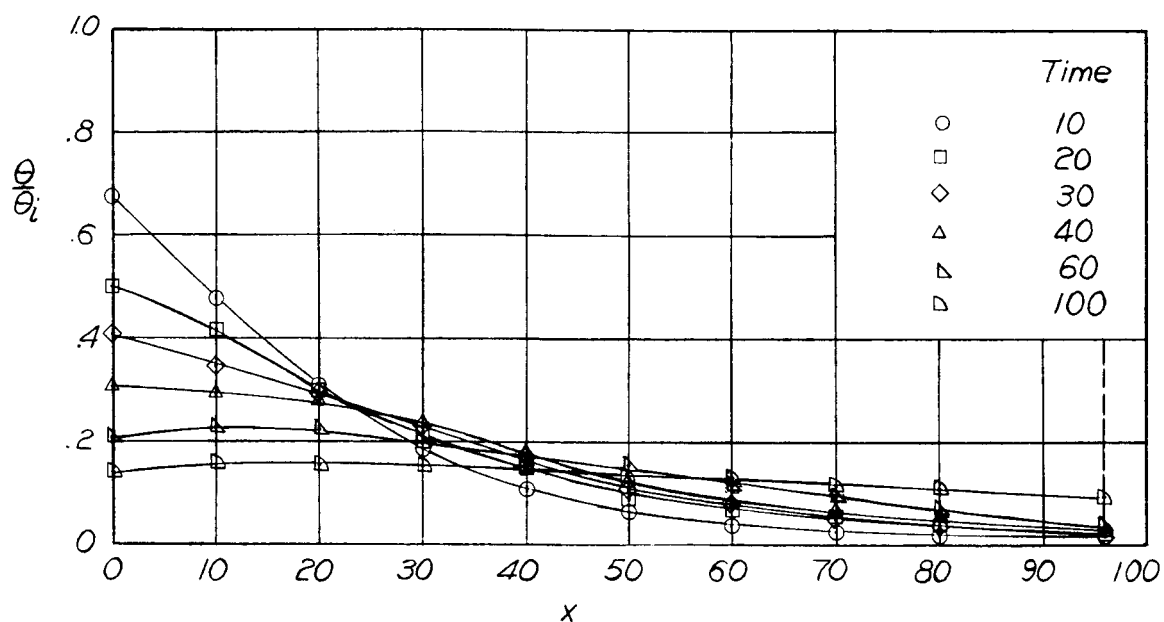
Experimental values of $\frac{\theta}{\theta_i}$.
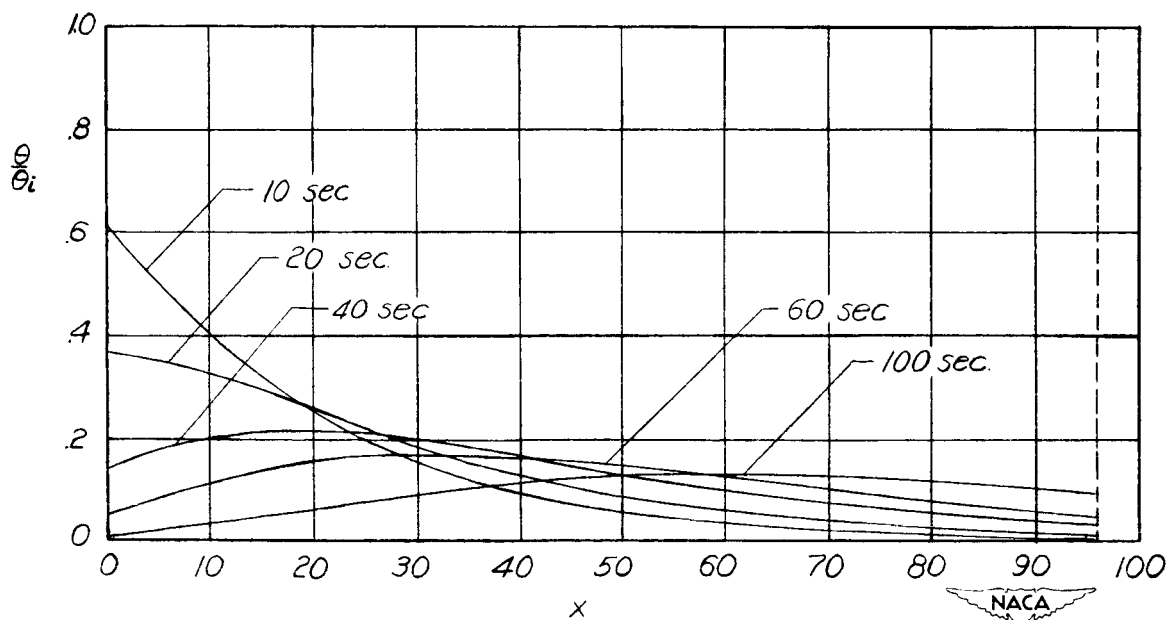
Computed values of $\frac{\theta}{\theta_i}$.

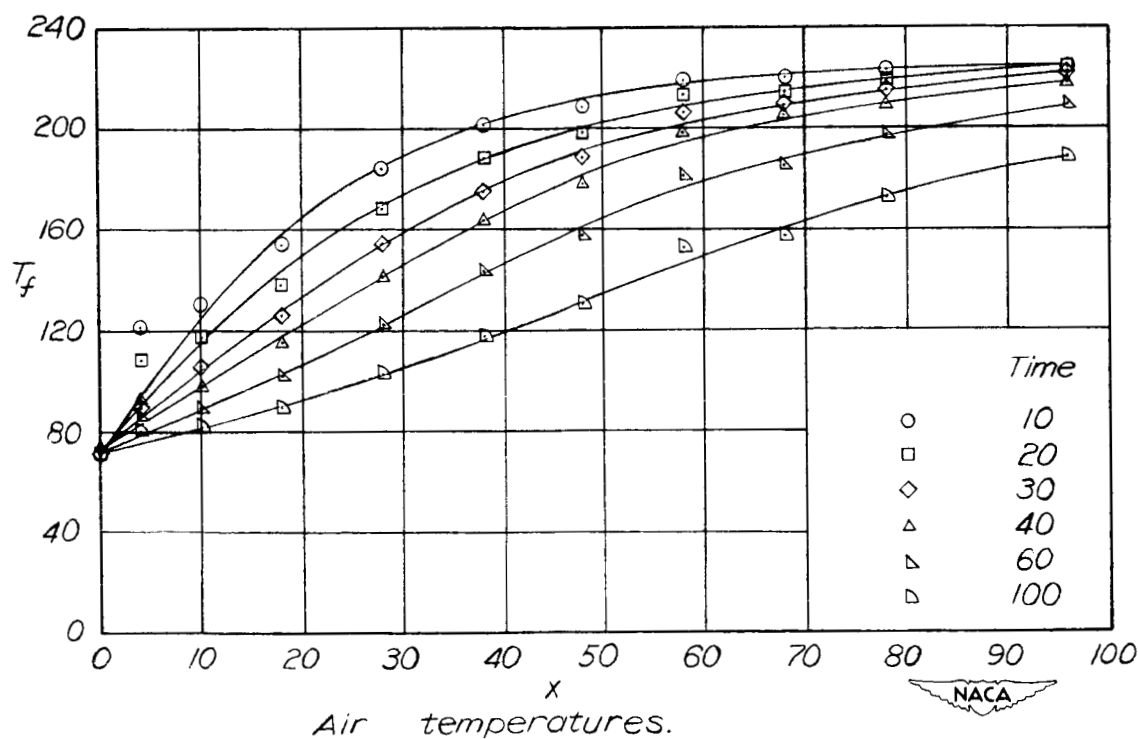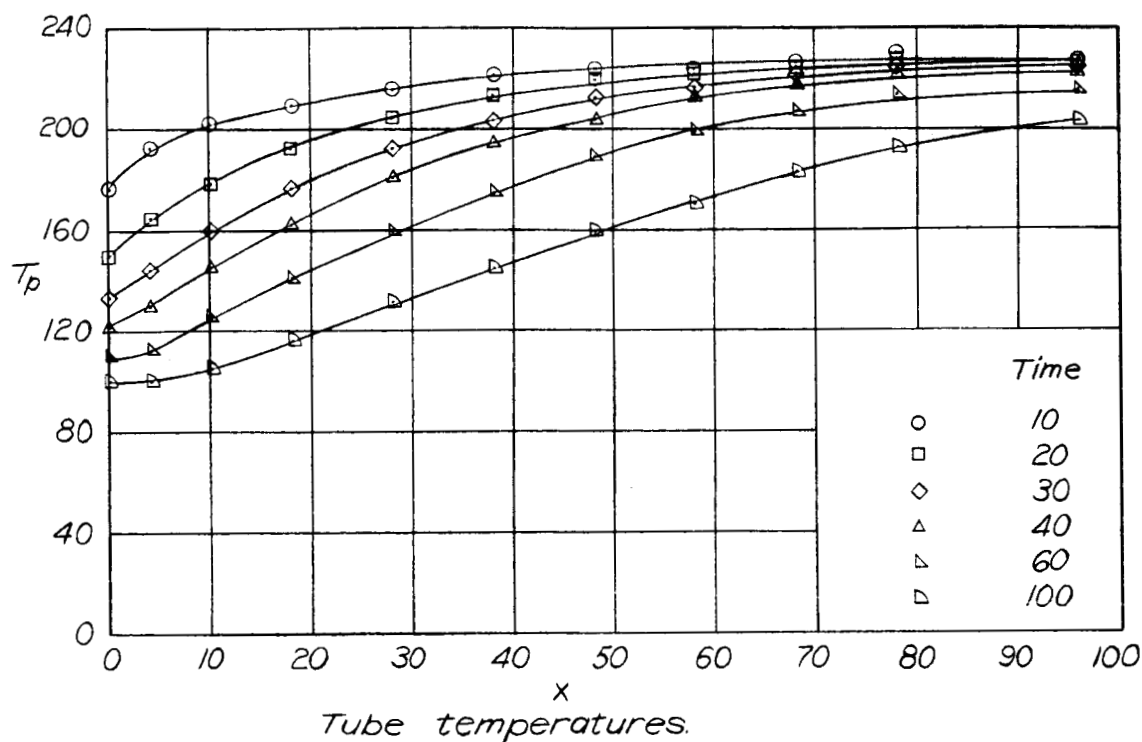(c) Initial tube temperature of 132°.
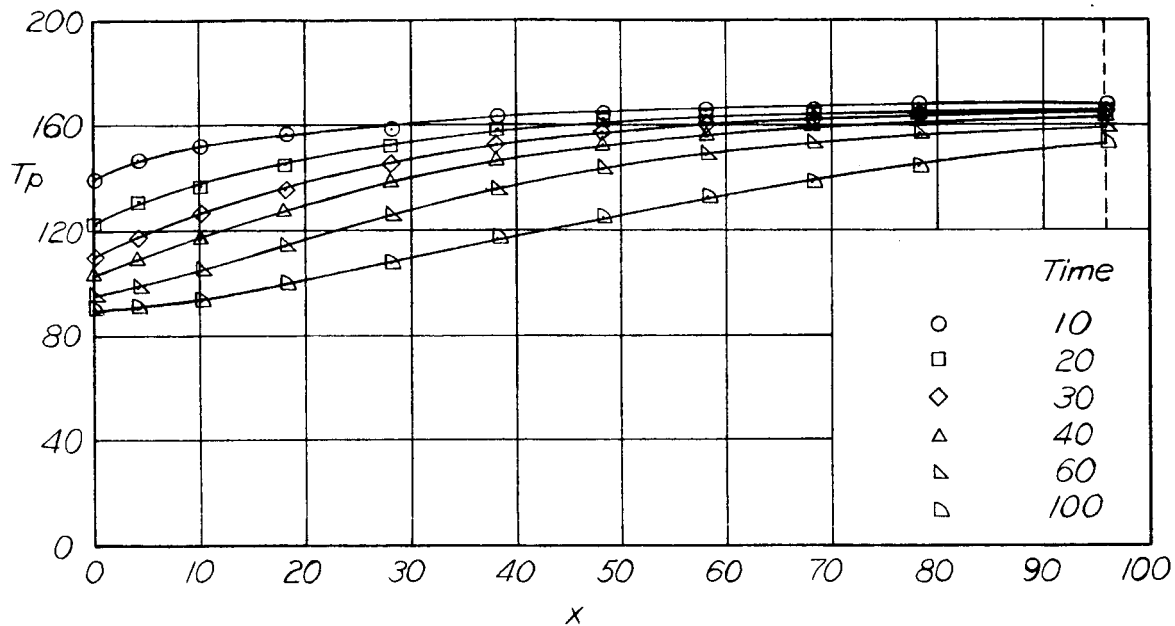
Figure 7.- Concluded.

240

T_p

10 sec.
20 sec.

200

160

120

80

60 sec.
100 sec.

30 sec.
40 sec.

40

0

0   10   20   30   40   50   60   70   80   90   100
X

Tube   temperatures.

240

T_f

10 sec.
20 sec.

200

160

60 sec.

120

100 sec.

80

30 sec.
40 sec.

40

0

0   10   20   30   40   50   60   70   80   90   100
X

Air   temperatures.

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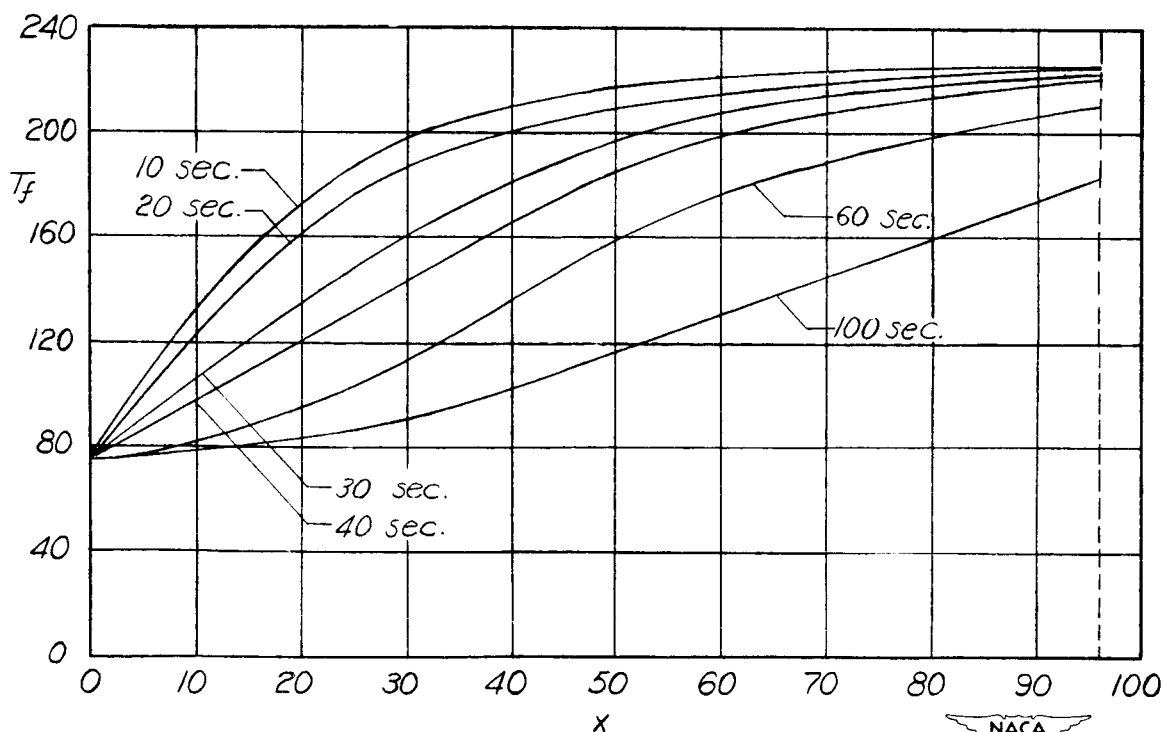
Figure 8.- Computed values of tube and air temperatures for an initial
tube temperature of 227°.

Tube temperatures.

Air temperatures.
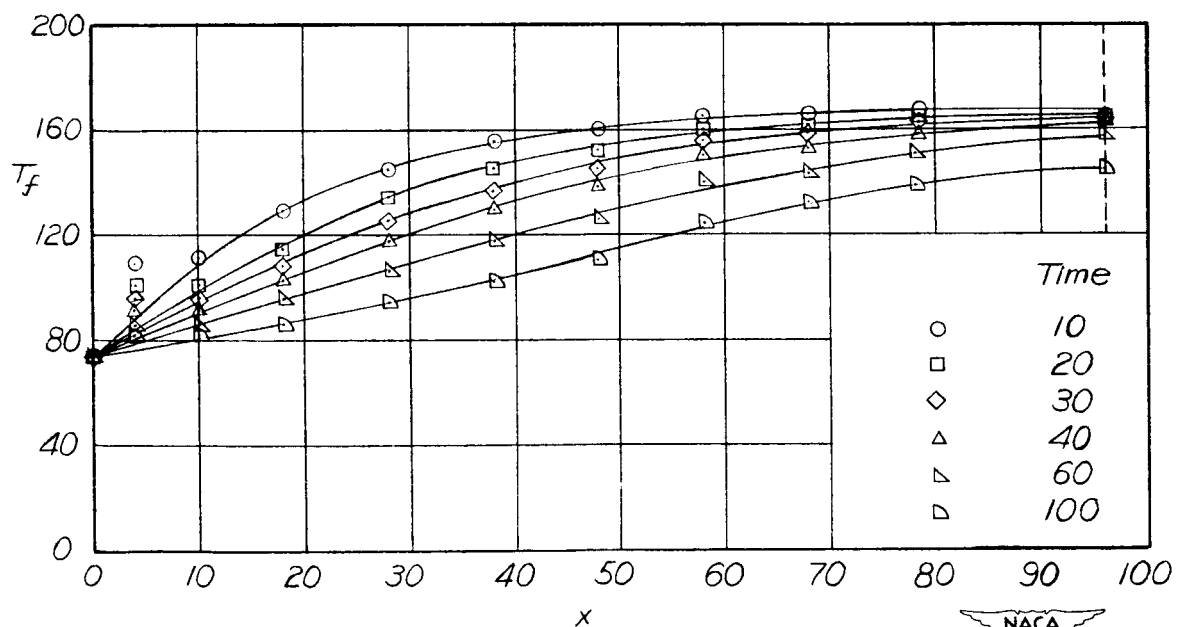
(a) Initial tube temperatures of 227°.

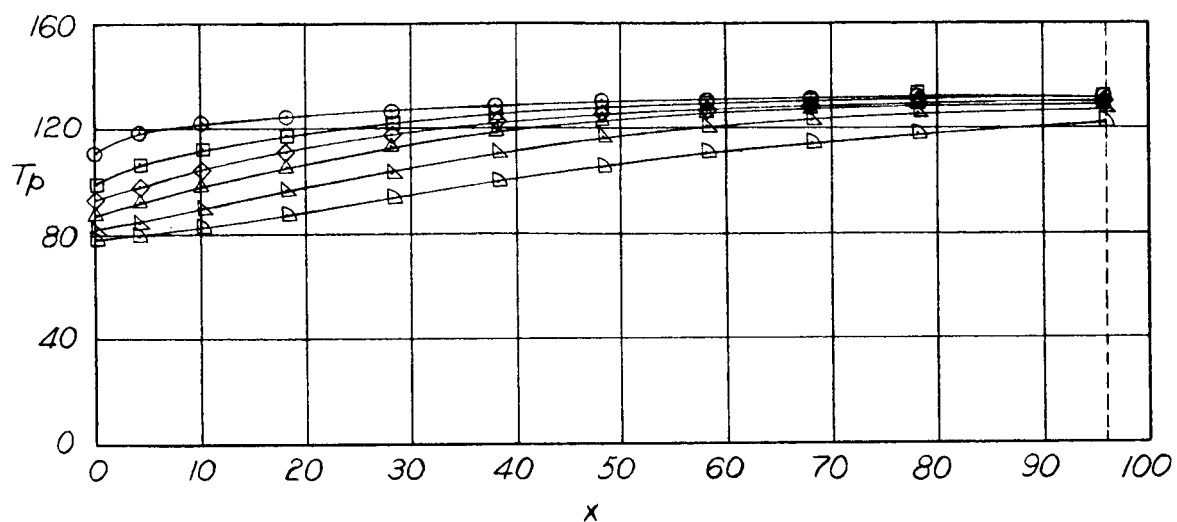Figure 9.- Experimental values of tube and air temperatures.

Tube temperature.

Air Temperature.

(b) Initial tube temperature of 167°.

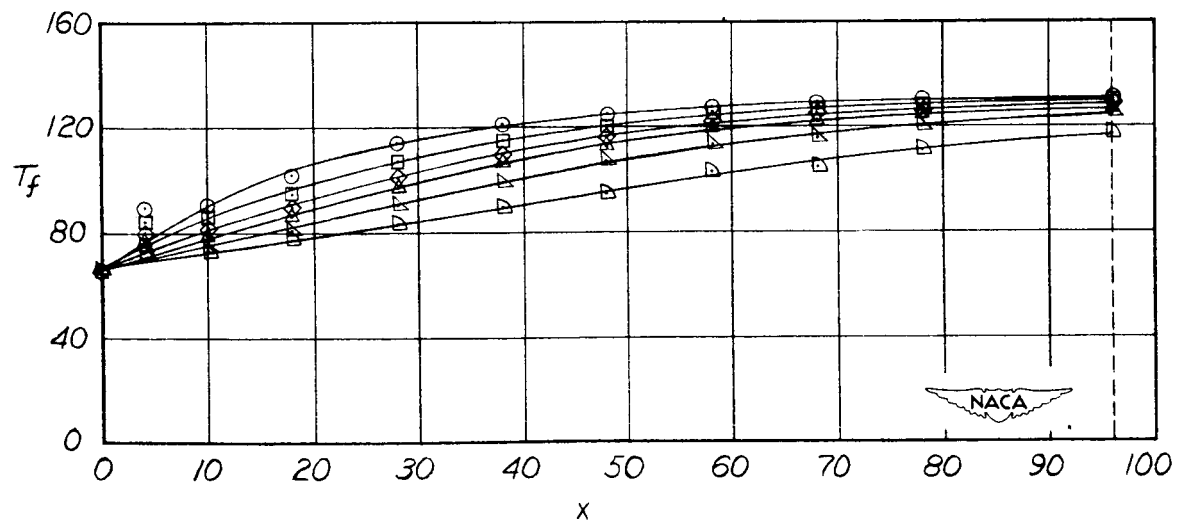Figure 9.- Continued.

Tube temperatures.

	Time
○	10
□	20
◇	30
△	40
◁	60
▷	100

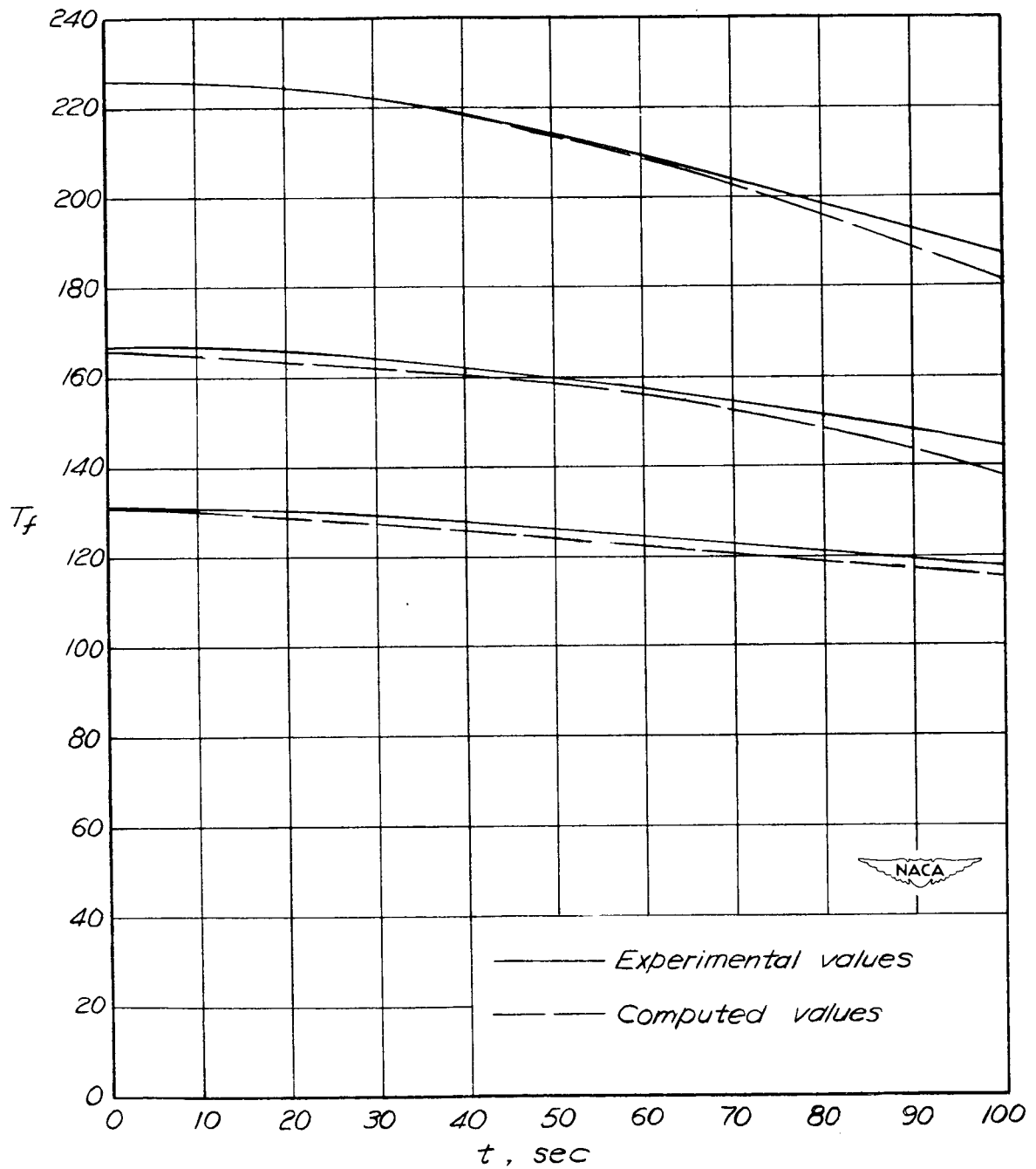Air temperatures.

(c) Initial tube temperature of 132°.

Figure 9.- Concluded.

Figure 10.- Variation of exit air temperature with time for initial tube temperatures of 132°, 167°, and 227°.